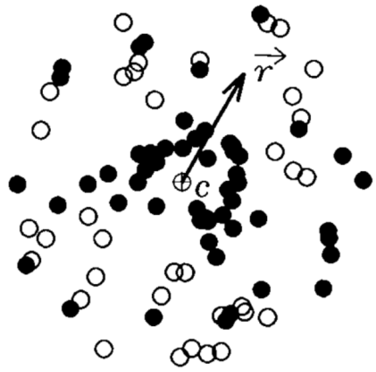
4. Hot Nuclear Matter Effects

There are two kinds of hot nuclear matter effects: the Debye screening and the (thermal) regeneration. The two affect the quarkonium production in an opposite way, and their balance controls the final state quarkonia.

4.1 Debye Screening



Suppose we put a pair of $Q\bar{Q}$ in a soup of light quarks and gluons, ***the string tension $\sigma(T)$ which controls the long-range force is strongly reduced*** *by the mean field in the deconfinement phase, $\sigma(T) \to 0$ for $T > T_c$. On the other hand, the input of $Q\bar{Q}$ pair will change the original charge distribution. The charge rearrangement leads to the Debye screening, namely the charge density around Q seen by $\bar{Q}$ becomes small, and* ***the Coulomb potential becomes a Yukawa potential***

$$-\frac{\alpha_c}{r} \to -\frac{\alpha_c}{r} e^{-r/r_D}$$

where r_D is the Debye screening length. ***When the screening length is shorter than the distance between Q and $\bar{Q}$, $\bar{Q}$ can not see Q, and the bound state disappears.*** This is the picture of Debye screening.

The screening length is inversely proportional to the temperature of QGP,

$$\lambda_D = \begin{cases} \sqrt{\frac{6}{g_q e_q^2}} \frac{1}{T}, & \text{Abelian approximation} \\ \frac{1}{\sqrt{\left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2}} \frac{1}{T}, & \text{pQCD with colored gluons} \end{cases}$$

We can estimate the Debye screening temperature in the frame of quantum mechanics *(C.Y.Wang, Introduction to High-Energy Heavy-Ion Collisions, World Scientific, 1994)*.

At $T > T_c$*, the Hamiltonian of the* $Q\bar{Q}$ *system in the rest frame of the bound state*

$$H = \frac{p^2}{m_Q} - \frac{\alpha_c}{r} e^{-r/r_D}$$

From the uncertainty relation

$$p^2 r^2 \sim 1$$

the average energy of the $Q\bar{Q}$

$$E = \frac{1}{m_Q r^2} - \frac{\alpha_c}{r} e^{-r/r_D}$$

The stable condition of the system $\frac{dE}{dr} = 0$ *leads to*

$$-\frac{2}{m_Q r^3} + \frac{\alpha_c \left(1 + \frac{r}{r_D}\right) e^{-\frac{r}{r_D}}}{r^2} = 0$$

To have a solution, the Debye screening length should satisfy

$$\frac{2}{m_Q r_D \alpha_c} \leq 0.84$$

With the pQCD calculated Debye screening length r_D*, we have for* J/ψ *state*

$$T \leq 209 \text{ MeV}$$

That means $T_D = 209$ ***MeV (~1.4Tc) is the*** J/ψ ***dissociation temperature.***

4.2 Potential Model and Lattice Results

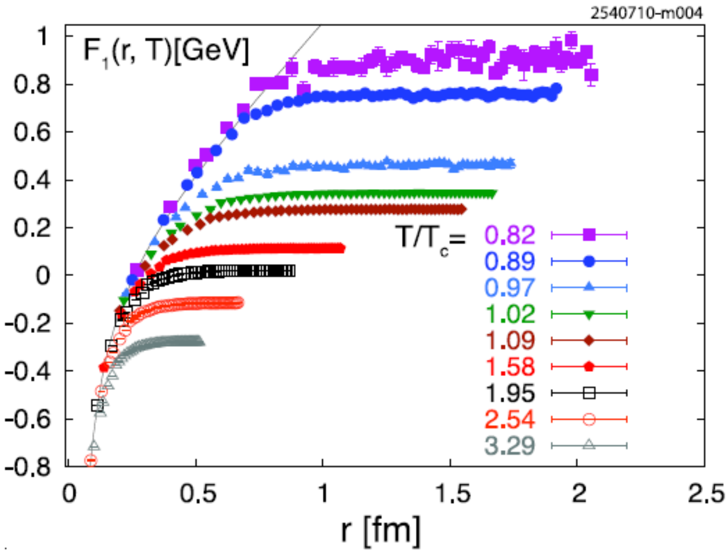
What is the heavy quark potential at finite temperature ?

The potential depends on the dissociation process in the medium.

For a rapid dissociation where there is no heat exchange between the heavy quarks and the medium, the potential is just the internal energy U***, while for a slow dissociation, there is enough time for the heavy quarks to exchange heat with the medium, the potential is the free energy F.***

From the thermodynamic relation $F = U - TS < U$*, the surviving probability of quarkonium states with potential* $V = U$ *is larger than that with* $V = F$*. In the literatures, a number of effective potentials in between* F *and* U *have been used (E.Shuryak and I.Zahed, Phys. Rev. D70, 054507(2004); C.Wong, Phys. Rev. C65, 034902(2002), C72, 034906(2005)).*

The free energy F can be extracted from the lattice calculations *(P.Petreczky, J. Phys. G37, 094009(2010))*.



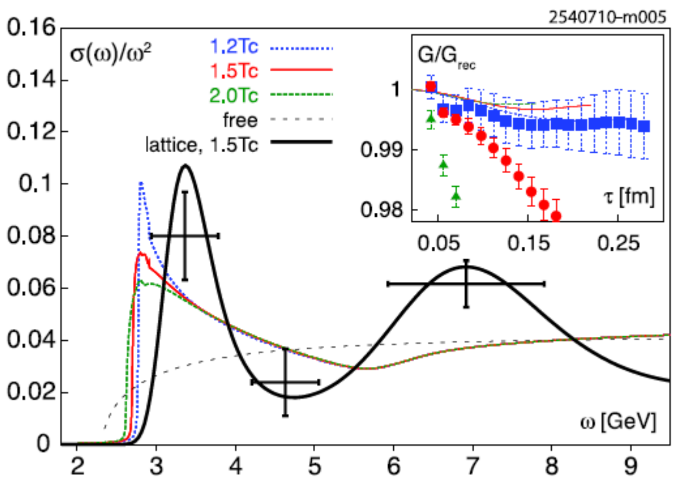
At T=0, F coincides with the Cornell potential.

At sufficiently short distances r, F is temperature independent.

At finite temperature, F is saturated at large r and the saturation becomes more and more fast with increasing temperature.

A constant F does not mean any interaction. The interaction range $r_I(T) <$ $r_{(Q\bar{Q})}$ means quarkonium dissociation.

The quarkonium dissociation can also be determined by spectral functions at finite temperature *(A. Mocsy and P.Petreczky, Phys. Rev. D77, 014501(2008))*.



The spectral function is clearly broadened at $T > T_c$. The J/ψ melting starts at (1.6-1.7)T_c *(M.Asakawa and T.Hatsuda, Phys. Rev. Lett. 92, 012001(2004)).*

Potential model calculation

Substituting the radial potential $V(r,T)$ into the Schroedinger equation, we obtain the binding energy $\varepsilon(T)$ and the average size $\langle r\rangle(T)$. The dissociation temperature is determined by

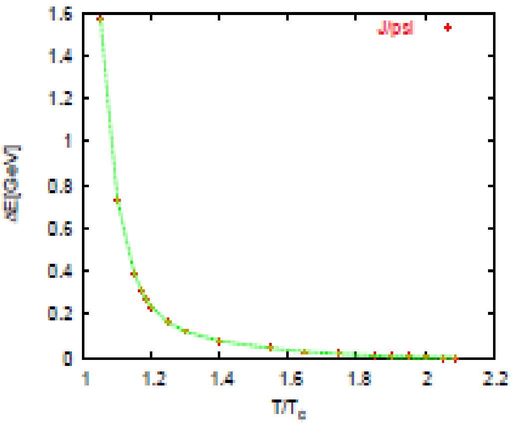
$$\epsilon(T_D) = 0$$

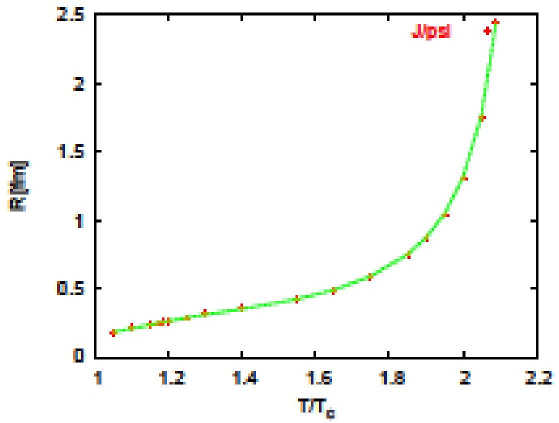
or equivalently

$$\langle r\rangle(T_D) = \infty.$$

For $V = U$ *(S.Digal, O.Kaczmarek, F.Karsch, H.Satz, Eur. Phys. J. C43, 71(2005))*,

state	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
T_d/T_c	2.10	1.16	1.12	> 4.0	1.76	1.60	1.19	1.17

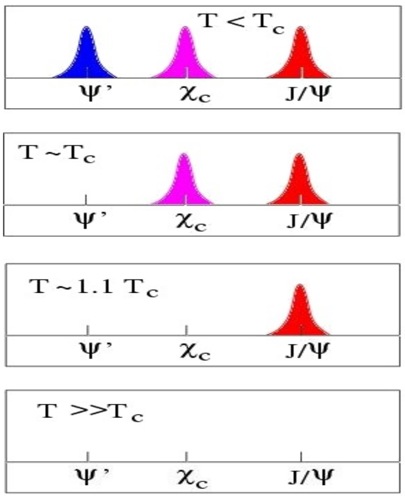




For $V = F$, T_D/T_c= 3, 1.1, 1 for $\Upsilon =$ 1s, 1p, 2s, and 1.2 for J/ψ.

Sequential suppression picture:

1) ***excited states are easy to be dissociated in comparison with ground state***
2) ***J/ψ is easy to be dissociated in comparison with Υ***
3) ***T_D at V=U is higher than that at V=F***

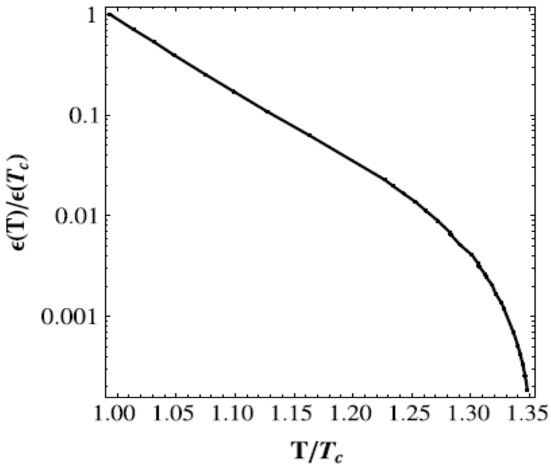


4.3 Relativistic Correction

Taking the covariant relativistic Schrodinger equations for the spin triplet (u_1^0, u_1^+, u_1^-) *and spin singlet* (u_0) *and the potential V = F = A + B (H.Satz, J. Phys. G 32, R25(2006))*

$$A(r,T) = -\frac{\alpha}{r}e^{-\mu r},$$
$$B(r,T) = \frac{\sigma}{\mu}\left[\frac{\Gamma(\frac{1}{4})}{2^{\frac{3}{2}}\Gamma(\frac{3}{4})} - \frac{\sqrt{\mu r}}{2^{\frac{3}{4}}\Gamma(\frac{3}{4})}K_{\frac{1}{4}}(\mu^2 r^2)\right] - \mu\alpha,$$

where Γ *is the Gamma function, K is the modified Bessel function of the second kind, and the screening mass* $\mu(T)$ *can be extracted from fitting the lattice simulated free energy (O.Kaczmarek, Eur. Phys. J. C 61, 811(2009)).*



In comparison with the non-relativistic calculation, the J/ψ dissociation temperature increases from 1.26Tc to 1.35Tc *(X.Guo, S.Shi and P.Zhuang, Phys. Lett. B718, 143(2012))*, ***the relativistic correction is 7%. The correction increases to 13% for V=U.***

Sequential melting:

The relativistic potential model can be used to estimate the flavor dependence of meson melting temperature *(S.Shi, X.Guo and P.Zhuang, Phy. Rev. D88, 014021(2013)).* ***The sequential melting temperature***

$$T_D(\pi) < T_D(D) \simeq T_D(\Phi) < T_D(J/\psi)$$

can explain the difference in meson elliptic flow observed in heavy ion collisions.

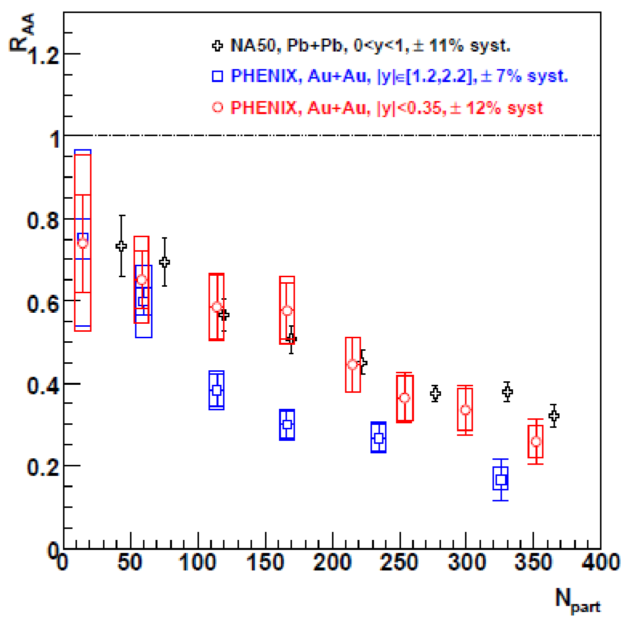
4.4 Quarkonium Regeneration

While charm quark number at SPS is small, there are more than 10 $c\bar{c}$ pairs produced in a central Au+ Au collision at RHIC, and the number is probably more than 200 at LHC *(R.V.Gavai et al., Int. J. Mod. Phys. A10, 3043(1995)).* ***These charm quarks in the QGP can be recombined into charmonium states.***

Therefore, there are two quarkonium production sources in high energy nuclear collisions: initial production and regeneration.

From RHIC data *(R.Cassagnac, J. Phys. G35, 104023(2008))* ***there are 2 puzzles:***

1) ***One is the same suppression at SPS and RHIC***
2) ***The other is the rapidity dependence.***



Since the fireball temperature is higher at RHIC energy and at central rapidity, the Debye screening induced suppression should be stronger at RHIC and at central rapidity.

While the cold nuclear matter effects may play a role here *(E.Ferreiro, F.Fleuret, J.Lansberg, et al., Phys. Lett., B680, 50(2009); B.Kopeliovich, I.Potashnikova, H.Pirner et al., Phys. Rev. C83, 014912(2011))*, ***the puzzles are direct hints for the introduction of quarkonium regeneration: more heavy quarks and in turn more regeneration at RHIC and at central rapidity .***

***Calculating the regeneration (and suppression) rate*:**

In the dynamical evolution of an inhomogeneous fireball, the dissociation is induced by inelastic collisions between quarkonia and constituents of the medium. At high energies, the gluon dissociation is the main dissociation process, and its inverse process is the main regeneration process:

$$Dissociation:\quad (Q\overline{Q}) + g \rightarrow Q + \bar{Q}$$

$$Regeneration:\quad Q + \bar{Q} \rightarrow (Q\bar{Q}) + g$$

Taking into account the fact that the distance between the two heavy quarks in vacuum is short, the potential between them is mainly the Coulomb part, the gluon dissociation can be approximately calculated by the Operator Production Expansion method (OPE, M.Peskin, Nucl. Phys., B156, 365(1979); G.Bhanot and M.Peskin, Nucl. Phys., B156, 391(1979)). To the leading order, the dissociation cross section can be analytically expressed as

$$\begin{aligned}
\sigma(1s) &= A\frac{(r-1)^{3/2}}{r^5}, \\
\sigma(1p) &= A\frac{(r-1)^{1/2}(9r^2-20r+12)}{r^7}, \\
\sigma(2s) &= A\frac{(r-1)^{3/2}(r-3)^2}{r^7},
\end{aligned}$$

with

$$\begin{aligned}
A &= \frac{2^{11}\pi}{27\sqrt{m_Q^3\epsilon(1s)}}, \\
r &= \frac{\omega}{\epsilon}.
\end{aligned}$$

where ω is the gluon energy in the quarkonium rest frame, and ε is the $Q\bar{Q}$ binding energy.

From the detailed balance, namely the same transition probabilities for the two processes, the cross section for the ground state regeneration is

$$\sigma_{Q\bar{Q}\rightarrow(1s)g}(s) = \frac{4}{3}\frac{(s-m_{Q\bar{Q}}^2)^2}{s(s-4m_Q^2)}\sigma_{(1s)g\rightarrow Q\bar{Q}}(s),$$

The cross sections at finite temperature

The OPE method to derive the above dissociation cross section is valid in vacuum and approximately valid at low temperature. At high temperature, especially when we reach the dissociation temperature T_D, the OPE fails to calculate the gluon dissociation. To describe the process in the medium, we consider the geometric relation between the integrated cross section and the size of the quarkonium *(L.Yan, N.Xu and P.Zhuang, Phys. Rev. Lett. 97, 232301(2006)),*

$$\sigma_{diss}(T) = \frac{\langle r^2 \rangle_{(Q\bar{Q})}(T)}{\langle r^2 \rangle_{(Q\bar{Q})}(0)} \sigma_{diss}(0)$$

where the averaged size of the quarkonium at finite temperature can be derived from the potential model.

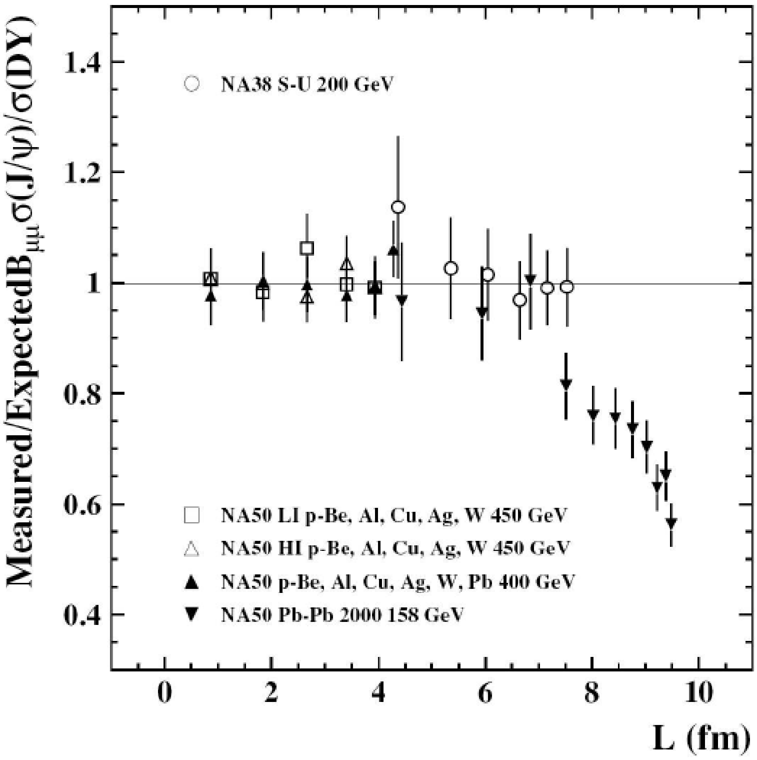
Besides the gluon dissociation process, other dynamical dissociation and regeneration processes are considered in literatures, for instance the quasi-free processes $(Q\bar{Q}) + p \to Q + \bar{Q} + p$ with p being gluons and quarks, which is probably important around the dissociation temperature *(L.Grandchamp and R.Rapp, Nucl. Phys. A709, 415(2002)).*

5. Heavy Flavor in A+A Collisions

Let us first review the history of J/ψ suppression, and then focus on the recent development at RHIC and LHC energies.

5.1 Normal and Anomalous Suppression at SPS

In p+A and light nuclear collisions at SPS, there is no QGP formation, and the J/ ψ suppression can well be explained by nuclear absorption (normal suppression). However, the Pb+Pb collisions show that the suppression of J/ψ (and ψ') in semi-central and central collisions goes beyond the normal absorption *(P.Cortese et al. [NA50 Collaboration], J. Phys. G31, S809 (2005)). This phenomenon, called anomalous J/ψ suppression, is considered as one of the most important experimental results in relativistic heavy ion collisions at SPS (U.W. Heinz and M.Jacob, Evidence for a new state of matter: An assessment of the results from the CERN lead beam programme," arXiv: nucl-th/0002042).*



The anomalous J/ψ suppression at SPS. The normal suppression is considered as a background.

Theory 1: Sequential melting *(Matsui and Satz)*

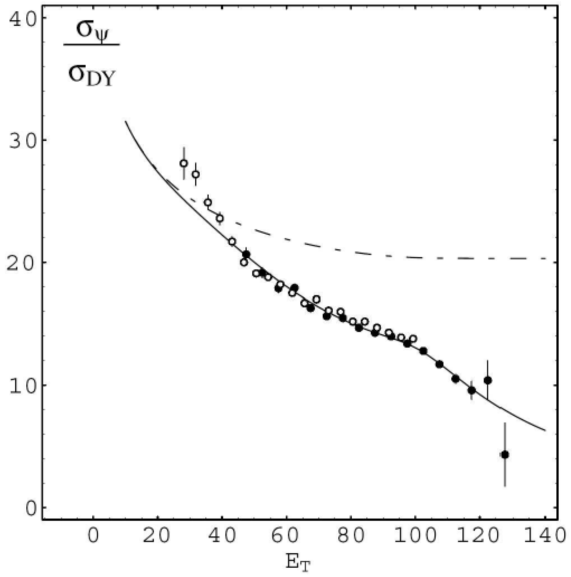
Considering the fact that about 40% of the final state J/ψ's come from the decay of ψ' and χ_c, the anomalous J/ψ suppression in Pb+Pb collisions at SPS is associated with the melting of ψ' and χ_c in the produced fireball.

Theory 2: Threshold model *(J.P.Blaizot and J.Y.Ollitrault, Phys. Rev. Lett. 77, 1703(1996))*

the J/ψ surviving probability

$$S_{J/\psi}(b) = \int d^2\mathrm{s} S_{J/\psi}^{nucl}(b,\mathrm{s})\Theta(n_c - n_p(b,\mathrm{s})),$$

where $S_{J/\psi}^{nucl}(b,s)$ is the J/ψ survival probability after the nuclear absorption, the density $n_p(b,s)$ is proportional to the energy density of the matter. When n_p is larger than a critical value n_c, all the J/ψ's are absorbed by the matter, and those J/ψ's outside the region suffer only normal suppression. The threshold density n_c is a parameter.



The J/ψ suppression at SPS, the solid and dashed lines are from the threshold model with and without anomalous suppression.

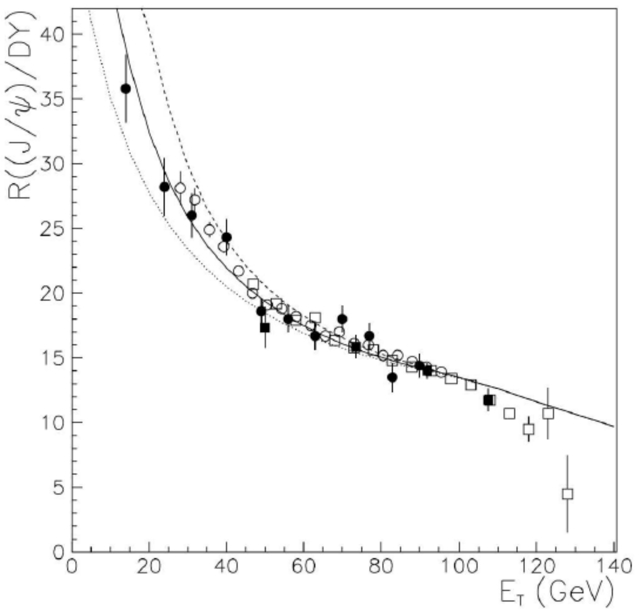
Theory 3: Comover model (*A.Capella, E.J.Feireiro and A.B.Kaidalov, Phys. Rev. Lett. 85, 2080(2000)*)

Not only partons in the deconfined phase can induce anomalous suppression, but also the secondary particles like π, ρ, ω mesons (so-called comovers) in a hot and dense hadron gas can interact with charmonia inelastically and cause J/ψ suppression *(J.Ftacnik, P.Lichard and J.Pisut, Phys. Lett. B207, 194(1988); S.Gavin, M.Gyulassy and A.Jackson, Phys. Lett. B207, 257(1988), R.Vogt, M.Prakash and P.Koch, Phys. Lett. B207, 263(1988)). The suppression due to the comover effect can be schematically expressed as*

$$S^{co}_{J/\psi} = e^{-\int d\tau \langle v\sigma_{co}\rangle \rho_{co}(\tau)},$$

where the comover density $\rho_{co}(\tau)$ is normally obtained through some kind of evolution mechanism of the matter (generally assumed to be of Bjorken hydrodynamics), and the cross section σ_{co} is an adjustable parameter in the calculation.

By adjusting the comover cross sections (and possibly other parameters, such as formation times), interactions at the hadron level can reproduce the SPS data of J/ψ suppression.



The J/ψ suppression at SPS, the lines are calculations in the comover model.

Theory 4: sQGP *(C.Young and E.Shuryak, Phys.Rev. C79, 034907(2009))*

In a weakly coupled QGP (wQGP), charm quarks would fly away from each other as soon as enough energy is available, ***while in a strongly coupled QGP (sQGP), the strong attraction between quarks opens the possibility of returning to the J/ψ ground state, leading to a substantial increase in surviving probability.*** *The charm quark motion in the medium is described by a Langevin equation,*

$$\frac{d\vec{p}}{dt} = -\gamma\vec{p} + \vec{\eta} - \vec{\nabla}V,$$

where η is a Gaussian noise variable, normalized as $\langle \eta_i(t)\eta_j(t)\rangle = 2m_c T$ with i,j indexing transverse coordinates, the drag coefficient in strongly interaction matter is large $\gamma = (2-4)(\pi T^2)/(1.5m_c)$, ***and V is the heavy quark potential (F<V<U).***

The surviving probability of charmonia in sQGP is larger than that in wQGP. This gives an alternative way to explain why there is no large difference between suppressions at SPS and RHIC.

5.2 Regeneration at RHIC and LHC

In A+A collisions at SPS energy, there is maximum of one $c\bar{c}$ pair produced in central Pb+Pb collision ($N_{c\bar{c}}$~0.2 at E_{lab}=158 A GeV). If the two quarks can not form a charmonium bound state close to their creation point, the probability to recombine in the medium and form a charmonium state is small and can probably be neglected.

However, for nuclear collisions at collider energies (RHIC and LHC), the situation becomes quite different. In a central Au+Au collision at the maximum RHIC energy, about 10-20 $c\bar{c}$ pairs are produced, and the uncorrelated c and $\bar{c}$ from different pairs have a significant probability (proportional to square of the number of $c\bar{c}$ pairs) to meet and form a charmonium bound state in the medium. The J/ψ regeneration in partonic and hadronic (or mixed) phase arises as a possible new mechanism for charmonium production in heavy ion collisions at RHIC and LHC.

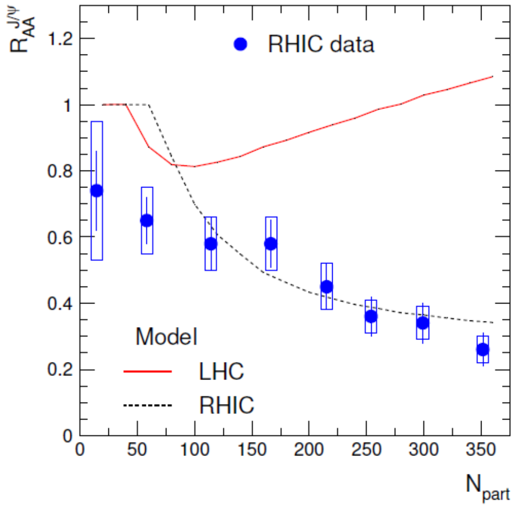
Regeneration model 1: Statistical production *(P.Braun-Munzinger, J.Stachel, Phys. Lett. B490, 196(2000))*

The entire charmonium production occurs statistically at the hadronization of the QCD matter. *The model is a direct extension of the thermal model which describes well the ratio of light hadron yields in relativistic heavy ion collisions.*

The nuclear modification factor

$$R_{AA}^{J/\psi} = \frac{dN_{J/\psi}^{AA}/dy}{N_{coll} \cdot dN_{J/\psi}^{pp}/dy}$$

from this model (P.Braun-Munzinger, nucl-th/0701093) reproduces well the J/ψ suppression in central Au+Au collisions at RHIC. ***At LHC energy $\sqrt{s_{NN}}$ = 5.5 TeV, the large charm production cross section results in an enhancement of J/ψ production for semi-central and central collisions.***



Regeneration model 2: Kinetic formation *(R.L.Thews, M.Schroedter, and J.Rafelski, Phys. Rev. C63, 054905 (2001))*

Unlike the statistical model where quarkonium production happens only at Tc, the quarkonia in the kinetic formation model can be regenerated continuously throughout the QGP region and suffer from dissociation processes. The competition between the formation and suppression is characterized by the kinetic equation *(R.L.Thews and M.L.Mangano, Phys. Rev. C73, 014904(2006))*

$$\frac{dN_{J/\psi}}{dt} = \lambda_F N_c N_{\bar{c}}/V(t) - \lambda_D N_{J/\psi}\rho_g$$

where ρ_g is the gluon density, and $\lambda = \langle \sigma v_{rel} \rangle$ is determined by the cross section and relative velocity, averaged over the momentum distribution of the initial particles.

Regeneration model 3: 2-component model *(L.Grandchamp and R.Rapp, Phys. Lett. B523, 60 (2001))*

A common assumption of the statistical hadronization model and the kinetic formation model is that the initially produced charmonia are entirely destroyed (or not formed) in the QCD medium. This is probably a good approximation for central heavy ion collisions at LHC energy, but for nuclear collisions at SPS and RHIC, as well as for peripheral collisions at LHC, we need to include initial production together with normal and anomalous suppressions.

The final yield of quarkonia is the sum of the direct and thermal production,

$$N_{J/\psi} = N_{J/\psi}^{dir} + N_{J/\psi}^{th}$$

At RHIC energy, the regeneration and initial production become equally important in central collisions *(R.Rapp, D.Cabrera and H.van Hees, arXiv:nucl-th/0608033)*.