Hanbury Brown-Twiss (HBT) interferometry with respect to the triangular flow-plane

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Background and Motivation

Hanbury-Brown–Twiss (HBT) interferometry (also, 'intensity interferometry' or 'femtoscopy') relies on two-particle momentum correlations to study the geometric and flow properties of heavy-ion collisions:

- azimuthally-sensitive HBT analyses communicate important information about deformations in the structure of the freeze-out surface
- odd harmonics present in HBT radii known to open the window to the study of event-by-event fluctuations
- fulfills a vital role in constraining the initial state of the fireball and its subsequent evolution

HBT Basics

Two particles: $\vec{p}_1,\ \vec{p}_2 \longrightarrow \vec{q} \equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$

Correlation function: $C(\vec{p}_1, \vec{p}_2) \equiv \dfrac{E_{p_1} E_{p_2} \frac{dN}{d^3p_1 d^3p_2}}{\left(E_{p_1} \frac{dN}{d^3p_1}\right)\left(E_{p_2} \frac{dN}{d^3p_2}\right)}$

Ignoring final-state interactions, C may be fit to the form:

$$C(\vec{q}, \vec{K}) = 1 \pm \lambda(\vec{K}) \exp\left(- \sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j\right),$$

$R^2_{ij} = R^2_{ij}(|\vec{K}|, \Phi_K) \rightarrow$ *measure* Φ_K *with respect to what?*

Fourier moments of $R^2_{ij}(\vec{K})$

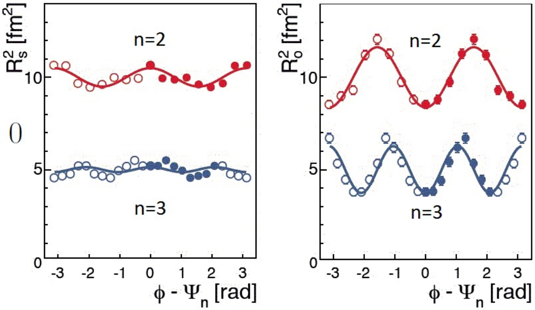
- Experimentally, one measures HBT correlations as a function of the *difference* between Φ_K and one of the flow angles Ψ_n
  $\Rightarrow$ we plot observable quantities against $\Phi_K - \Psi_n$, $(n = 1, 2, 3, \ldots)$
- The flow angle is defined by Ψ_n in $v_n e^{in\Psi_n} \equiv \left\langle e^{in\phi_p} \right\rangle$
- The v_n are the anisotropic flow coefficients and ϕ_p is the azimuthal angle of $\vec{p}_T$ of the emitted particles in the lab frame
- $\Rightarrow$ Fourier-decompose the R^2_{ij}:

$$\begin{aligned} R^2_{ij}(|\vec{K}|, \Phi_K) &= 2\sum_{n=1}^{\infty}\Big(R^{2(c)}_{ij,n}(|\vec{K}|)\cos[n(\Phi_K - \Psi_n)] \\ &+ R^{2(s)}_{ij,n}(|\vec{K}|)\sin[n(\Phi_K - \Psi_n)]\Big) + R^2_{ij,0}(|\vec{K}|) \end{aligned}$$

PHENIX data

T. Niida, (QM 2012, arXiv:1304.2876) (integrated over $K_\perp$)

$R^2_{s,n} > 0$



$R^2_{o,n} < 0$

Important features to understand:

- Different signs of Fourier coefficients in out and side directions
- Different oscillation amplitudes: $R^2_{o,n}/R^2_{s,n} \gg 1$

Emission function

We define the emission function $S(x,K)$ as the Wigner density of the fireball

$$\text{Emission function:} \quad \int d^4x\, S(x,K) = E_K \frac{dN}{d^3K}$$

Taking $\lambda(\vec{K}) = 1$, C and S may be related by

$$C(\vec{q},\vec{K}) \approx 1 + \left| \frac{\int d^4x\, e^{iq\cdot x} S(x,K)}{\int d^4x\, S(x,K)} \right|^2$$

- For Gaussian sources $S(x,K)$, $R^2_{ij} = \langle (\tilde{x}_i - \beta_i \tilde{t})(\tilde{x}_j - \beta_j \tilde{t}) \rangle$, where
- $\tilde{x}_i \equiv x_i - \langle x_i \rangle$, $\tilde{t} \equiv t - \langle t \rangle$, $\vec{\beta} \equiv \vec{K}/K^0$ and
- $\langle f(x) \rangle \equiv \frac{\int d^4x\, f(x) S(x,K)}{\int d^4x\, S(x,K)}$

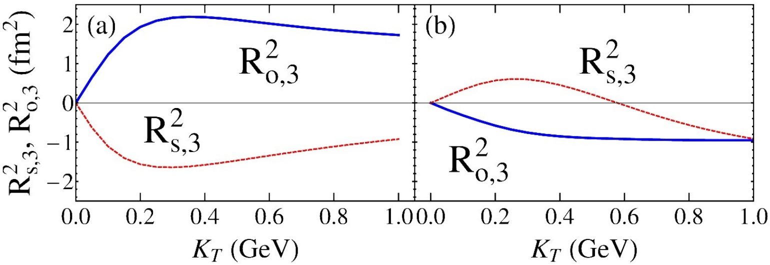
$\Rightarrow$ given $S(x,K)$, $R^2_{ij}(\vec{K})$ may be computed directly

Emission function

- Consider S with two kinds of different triangular deformations:
  - "Geometric case" - Triangular spatial deformation with radial flow, no triangular flow
  - "Flow case" - Triangular flow, no spatial deformation
- Can obtain triangular oscillations of R^2_{ij} from
  - triangular flow deformation
  - triangular spatial deformation coupled to radial flow
  - combinations thereof

HBT oscillation amplitudes: two examples

Geometric case | Flow case



Conclusions

- Without radial flow, a triangular spatial deformation of the source at freeze-out leaves no measurable trace in the HBT radii oscillations
- Triangular oscillations of HBT radii may generally result from an admixture of triangular collective flow *and* triangular spatial deformation coupling to radially symmetric flow
- We can distinguish "flow domination" from "geometry domination" by the phases and K_T-dependence of the respective oscillation amplitudes; PHENIX data appear to point to "flow domination"

Thanks for your attention!

Thanks also to my collaborators

Ulrich Heinz and Chun Shen!

Back-up slides

Double-Fourier formalism

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Define

$$S_{\ell,m} \equiv e^{-im\Psi_3} \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{i\ell\phi} \int_{-\pi}^{\pi} \frac{d\Phi_K}{2\pi} e^{im\Phi_K} S(\phi, \Phi_K),$$

$$\longrightarrow \mathcal{Z}_\ell \equiv e^{-i\ell\Psi_3} \sum_{m=-\infty}^{\infty} S_{\ell,m-\ell} e^{-im(\Phi_K - \Psi_3)} \equiv \mathcal{X}_\ell + i\mathcal{Y}_\ell$$

We can show, e.g.,

$$\langle x_s^2 \rangle = \int_{-\infty}^{\infty} d\eta \int_0^\infty \tau d\tau \int_0^\infty r\, dr \pi r^2 \left(\mathcal{X}_0 - \mathcal{X}_2\right)$$

$$\langle x_s \rangle = \int_{-\infty}^{\infty} d\eta \int_0^\infty \tau d\tau \int_0^\infty r\, dr 2\pi r \mathcal{Y}_1$$

- Since $R_s^2 = \langle x_s^2 \rangle - \langle x_s \rangle^2$, no dependence on $\ell \geq 3$ (similarly for other R_{ij}^2)!
- N.B.: same expression contains all orders in Φ_K

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Toy model for the source

$$
\begin{aligned}
S(x,K) &= \frac{S_0(K)}{\tau}\exp\left[-\frac{(\tau-\tau_f)^2}{2\Delta\tau^2}-\frac{(\eta-\eta_0)^2}{2\Delta\eta^2}\right. \\
&\quad -\frac{r^2}{2R^2}\left(1+2\bar{\epsilon}_3\cos(3(\phi-\bar{\psi}_3))\right) \\
&\quad \left. -\frac{M_\perp}{T_0}\cosh(\eta-Y)\cosh\eta_t+\frac{K_\perp}{T_0}\cos(\phi-\Phi_K)\sinh\eta_t\right]
\end{aligned}
$$

where

$$
\eta_t=\frac{\eta_f r}{R}\left(1+2\bar{v}_3\cos(3(\phi-\bar{\psi}_3))\right)
$$

- $\bar{\epsilon}_3$: triangular azimuthal deformation
- $\bar{v}_3$: triangular flow deformation
- η_f: collective radial flow rapidity
- $\bar{\psi}_3$: triangular flow velocity angle, points in direction of largest flow rapidity and steepest descent of spatial density profile (note: $\Psi_n \neq \bar{\psi}_n$ in general)